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Modeling Malaria Elimination in Nigeria: Modified SEIR-SEI Model with Intervention Effectiveness Analysis

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Abstract

Malaria remains a significant public health threat despite numerous interventions. This study proposes a modified SEIR-SEI model to improve malaria elimination strategies in Nigeria. The model incorporates Long Lasting Insecticide-treated bed Nets (LLINs), Artemisinin-based Combination Therapies (ACTs), Indoor Residual Spraying (IRS), Malaria Vaccines (MVs), and Environmental Sanitation (ES) alongside a compartment for individuals resistant to ACTs. We demonstrate the model's positivity, boundedness, and establish the disease-free equilibrium point. Stability analysis confirms the equilibrium's stability under certain conditions. The basic Reproduction number (R_0) of 0.124 indicates a gradual decline in malaria prevalence. Further analysis reveals that combining all interventions minimizes mosquito breeding and R_0 decreases as intervention rates increase, reaching 0.0441 with maximized intervention usage, suggesting potential malaria elimination in Nigeria. This model provides valuable insights for optimizing intervention strategies and achieving malaria elimination in Nigeria.

Keywords: Malaria; Compartmental model; Basic reproduction number; SEIRR-SEI

Introduction

Malaria is a contagious disease caused by parasites called *Plasmodium*. The bites of infected female *Anopheles* mosquitoes transmit these parasites to humans. Malaria remains a major public health concern around the world. It is one of the main causes of mortality and sickness in many underdeveloped nations, particularly among children and pregnant women [1]. Malaria is widespread in more than 100 nations and territories worldwide, largely in the less developing tropical areas of Africa, Asia, and Latin America [2]. Malaria cases were estimated at 241 million in 2020, with 627,000 deaths directly related to the disease, 95% of which occurred in Africa. Nigeria and most of West Africa are currently classified as being in the control phase, according to the global malaria elimination program classification [3]. Millions of people in Africa still do not have accessibility to measures that prevent malaria such as insecticide-treated bed nets, Indoor Residual Spraying (IRS), and proper medical care with ACTs drugs, among others. In a bid to lessen the impact of malaria, the World Health Organisation (WHO) initiated the Roll Back Malaria (RBM) campaign to halve malaria fatalities by 2010 and reduce them again by 2015.

In 2019, a new malaria vaccine for infants was suggested by the WHO (2022) after effective trials in Ghana, Kenya, and Malawi. This vaccine is anticipated to boost the resistance against infection caused by malaria, in addition to the other preventive and treatment measures already in place. However, it only showed limited efficacy in reducing malaria cases in young children [4,5].

Numerous significant studies by different researchers have explored diverse mathematical models related to malaria. These models have been employed to address the escalation of malaria and to propose strategies for managing malaria in regions significantly affected by outbreaks across the globe. Kumar et al. [6] introduced a new fractional SIRS-SI model to describe the transmission of malaria. The model suggested the use of vaccines, antimalarial medicines, and spraying as treatment and control measures for malaria. The possibility of solutions and their distinctiveness in the fractional SIRS-SI model were examined. The study demonstrated that treatments have a significant impact on the dynamics between humans and vector populations. Numerical simulation results to illustrate the effects of different treatment parameters on malaria showed that the proposed approach is accurate and effective in combating the disease.

Anickode [7] utilized a basic compartmental model for malaria transmission dynamics to evaluate different strategies for controlling malaria. The numerical simulations of the model

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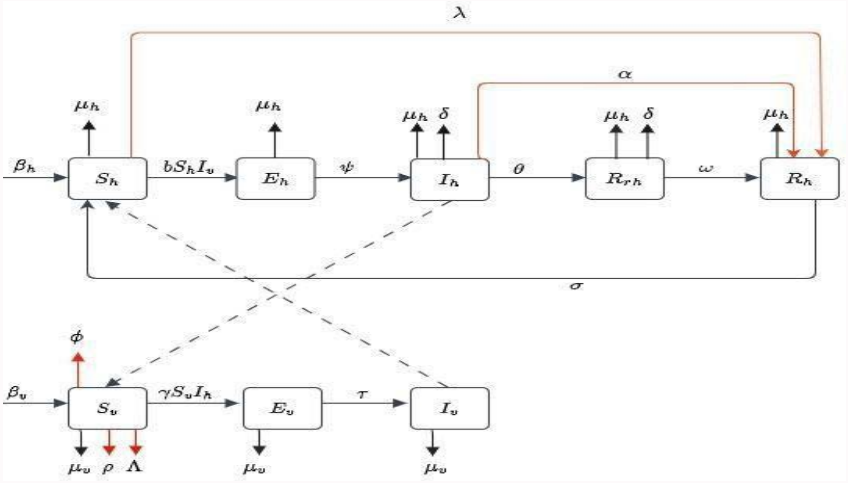

Figure 1: A modified schematic diagram for malaria transmission.

indicated that relying solely on a vaccine as an anti-malaria strategy might not be sufficient to eliminate malaria in areas where the disease is prevalent. However, the study suggested that combining vaccination with other strategies for managing disease vectors, particularly those based on insecticides to reduce mosquito populations, can lead to the elimination of malaria. Hakizimana et al. [8] studied the dynamics of malaria transmission using the nonlinear forces of infections. The study examined the SEIR-SEI mathematical model for human and mosquito populations but failed to incorporate prevention and control strategies to reduce malaria in the model. Optimal control strategies were used to reduce malaria transmission rates using three control variables and the Caputo fractional derivative. The numerical simulation results showed that the use of three control measures, namely ITNs, IRS, and ACTs together, is much more effective in reducing the number of sick people and infected mosquitoes in endemic areas. Also, the simulation showed that the number of recuperated humans would rise over the next few years.

Segun et al. [9] proposed a deterministic compartmental model to study the dynamics of malaria spread in a population and then incorporated three control measures: treated bednets, medication treatment, and insecticide spray. The reproduction number obtained for both the disease-free and the endemic equilibria was found to be asymptotically stable. The study focused on an optimal control problem to examine and compare different control strategies for eradicating malaria. The numerical results of the study demonstrated that the combined use of three control measures-treated bednets, medication, and insecticide spray-has the greatest effectiveness in controlling and managing the spread of malaria in the population. In addition, Kobe [10] suggested a deterministic mathematical model with five state variables for the human population and two state variables for the mosquito population to better comprehend the transmission and management of the spread of malaria. In order to create the SPITR model, the model added protected and treatment classes to the fundamental SIR model. While the treatment class offered two treatment techniques-the treatment itself and Intermittent Preventive Treatment of malaria in Pregnancy (IPTP)-the protected class introduced two intervention measures, ITN and IRS. The findings of the numerical analysis and simulations indicated that it is possible to effectively control malaria by combining strategies for prevention and treatment to decrease the population of affected people and mosquitoes. The study did not consider other control measures and treatment strategies that can help reduce the spread of malaria.

Model Formulation

A modified SEIRR-SEI mathematical model was developed using ordinary differential equations. In this model, the population is divided into eight compartments. The fundamental model has coupling compartments for human population and host population. For the human population, we have Susceptible (S_h) which denotes the malaria free individuals, exposed (E_h) which denotes individuals who are infected by malaria without the ability to transmit the disease, Infected (I_h) denotes individuals who are infected with malaria with the ability to transmit the disease, (R_{rh}) denotes individuals with drug resistance symptoms and Recovered (R_h) denotes individuals who recovered from malaria. For the vector population, we have Susceptible (S_v), Exposed (E_v), and Infectious (I_v) (Figure 1 and Table 1).

The total size of the human population is expressed as:

$$N_h = S_h + E_h + I_h + R_{rh} + R_h \tag{1}$$

The total size of the host or malaria population is expressed as:

$$N_v = S_h + E_v + I_v \tag{2}$$

Model Equations

$$\frac{dS_h}{dt} = \beta_h + \sigma R_h - (\mu_h + \lambda) S_h - bS_h I_v$$

$$\frac{dE_h}{dt} = bS_h I_v - (\mu_h + \psi) E_h$$

$$\frac{dI_h}{dt} = \psi E_h - (\mu_h + \alpha + \theta + \delta) I_h$$

Table 1: Description of parameters of the model.

Parameters	Description
β_h	recruitment rate of human population
b	transmission rate of human
γ	transmission rate of vector
ψ	rate movement of humans from the Exposed class to the Infected class
α	treatment rate of infected humans with ACTs
σ	loss of immunity of the human population
δ	disease induced mortality rate of human
μ_h	natural mortality rate of human
β_v	recruitment rate of the vector
μ_v	natural mortality rate of vector
θ	rate of movement of infected humans to resistance class
ω	recovery rate of infected humans that are resistance to Anti-Malaria Drugs (ACTs)
λ	recovery rate of susceptible humans with the use of malaria vaccines
τ	rate of movement of vector from the Exposed class to the Infected class
ρ, ∅, Λ	Prevention efforts that lead to the death or reduction in the number of vectors that is, Indoor Residual Spraying (IRS), Long Lasting Treated Insecticides bed nets (LLINs), and Environmental Sanitation respectively.

$$\begin{aligned}
\frac{dR_{rh}}{dt} &= \theta I_h - (\mu_h + \omega + \delta) I_{rh} \\
\frac{dR_h}{dt} &= \omega R_{rh} + \alpha I_h - (\sigma + \mu_h) R_h + \lambda S_h \\
\frac{dS_v}{dt} &= \beta_v - \gamma S_v I_h - (\mu_v + \rho + \phi + \wedge) S_v \\
\frac{dE_v}{dt} &= \gamma S_v I_h - (\mu_v + \tau) E_v \\
\frac{dI_v}{dt} &= \tau E_v - \mu_v I_v
\end{aligned} \tag{3}$$

Properties of the Model

Positivity of the solutions

Since the model above deals with human and mosquito populations, then every of its parameters are non-negative. Therefore, the positivity theorem holds.

Theorem

The solutions of the model equation (3) with the initial condition are non-negative for every time $t>0$. It then follows that:

If the initial condition is given as:

$S_h \geq 0, E_h \geq 0, I_h \geq 0, R_{rh} \geq 0, R_h \geq 0, S_v \geq 0, E_v \geq 0, I_v \geq 0$

Then the solution set/region $\{S_h, E_h, I_h, R_{rh}, R_h, S_v, E_v, I_v\}$ (t) of the system of equations (3) is positive (non-negative) for every time $t>0$.

Proof

To show that every solution of the equation is non-negative, each of the differential equation is considered separately.

From equation (2), we have:

$$\frac{dS_h}{dt} = \beta_h + \sigma R_h - (\mu_h + \lambda) S_h - b S_h I_v$$

Since $\beta_h + \sigma R_h$ is positive, thus, we drop it. The equation is then expressed as an inequality i.e.

$$\frac{dS_h}{dt} \geq -(\mu_h + \lambda + bI_v) S_h \tag{4}$$

Using the separation of variables and integrating both sides gives:

$$\begin{aligned}
\int \frac{dS_h}{S_h} &\geq -\int (\mu_h + \lambda + bI_v)\, dt \\
\int \frac{1}{S_h} dS_h &\geq -\int (\mu_h + \lambda + bI_v)\, dt \\
\ln S_h &\geq -\int (\mu_h + \lambda + bI_v) t + c
\end{aligned} \tag{5}$$

Take the exponential of both sides

$$S_h(t) \geq e^{-(\mu_h + \lambda + bI_v)t} \times e^c$$

Let $e^c = constant = k$, we have,

$$S_h(t) \geq ke^{-(\mu_h+\lambda+bI_v)t} \quad (6)$$

Using the initial condition:

$t = 0$, $S_h(0) \geq 0$

$$S_h(t) \geq S_h(0)e^{-(\mu_h+\lambda+bI_v)t}$$

Where $k = S_h(0)$

$$\geq S_h(0)e^{-(\mu_h+\lambda+bI_v)t} \geq 0$$

Since the initial population size of susceptible human is positive, i.e. $S_h \geq 0$, and the exponential function is always non-negative. It can then be concluded that $S_h(t)$ is positive.

From the second equation,

$$\frac{dE_h}{dt} = bS_hI_v - \mu_hE_h - \psi E_n$$

Since the term $\frac{bS_hI_v}{N_h}$ is positive, thus we drop it. The equation is then expressed as an inequality i.e.

$$\frac{dE_h}{dt} \geq -(\mu_h+\psi)E_h \quad (7)$$

Separating the variables and then integrate both sides to give

$$\int \frac{dE_h}{E_h} \geq -\int(\mu_h+\psi)dt$$

$$\int \frac{1}{E_h}dE_h \geq -\int(\mu_h+\psi)dt$$

$$\ln E_h \geq -\int(\mu_h+\psi)\,t + c \quad (8)$$

Take the exponential of both sides

$$E_h(t) \geq e^{-(\mu_h+\psi)t} \times e^c$$

Let $e^c = constant = k$, we have,

$$E_h(t) \geq ke^{-(\mu_h+\psi)t}$$

Using the initial condition:

$t = 0$, $E_h(0) \geq 0$

$$E_h(t) \geq E_h(0)\, e^{-(\mu h+\psi)t}$$

Where $K = E_h(0)$

$$\geq E_h(0)\, e^{-(\mu h+\psi)t} \geq 0 \quad (9)$$

Since the initial population size of exposed human is positive, i.e. $E_h \geq 0$, and the exponential function is always non-negative, then $E_h(t)$ is positive.

Similarly, it can be shown that the remaining equations of the model in equation (3) are positive for every $t>0$.

Boundedness of the solutions

The boundedness of a solution means that the solution of a mathematical model is within a specified range.

Considering the total population for both human (host) and mosquito (vector) of the model below:

$$N_h = S_h + E_h + I_h + R_{rh} + R_h$$

$$N_v = S_h + E_v + I_v$$

Their respective differential equations are given as:

$$\frac{N_h}{dt} = \frac{S_h}{dt} + \frac{E_h}{dt} + \frac{I_h}{dt} + \frac{R_{rh}}{dt} + \frac{R_h}{dt} = \beta_h - \mu_hN_h - \delta I_h - \delta I_{rh} \quad (10)$$

and

$$\frac{N_v}{dt} = \frac{S_v}{dt} + \frac{E_v}{dt} + \frac{I_v}{dt} = \beta_v - \mu_vN_v - (\rho+\phi+\wedge)S_v \quad (11)$$

All state variables are assumed to be positive since the model is dealing with population.

Theorem

The solutions of equation (3) are feasible for every $t>0$ if they enter the invariant region

$\pi = \pi_h \times \pi_v$

Proof

Let $\pi_{\ddot{u}\ddot{u}\ddot{u}\ddot{u}} = (S, E, I, R, R) \in \mathbb{R}_+^5$ be any solution of equation (3) with positive initial conditions. Assuming there is no death rate due to malaria or malaria does not kill i.e. δ=0

Then equation (10) becomes

$$\frac{dN_h}{dt} \le \beta_h - \mu_h N_h$$

$$\frac{dN_h}{dt} + \mu_h N_h \le \beta_h \tag{12}$$

Using the first order linear differential equation of the form $y' + p(t)y = q(t)$ where

$p(t) = \mu_h$ and $q(t) = \beta_h$

Then for the Integrating Factor (IF) for equation (12) is given as

$e^{\int \ddot{u}\ddot{u}(\ddot{u}\ddot{u})} = e^{\int \mu_h} = e^{\mu_h t}$

Multiply both sides of equation (12) by $e^{\mu h t}$ we have:

$$\frac{dN_h}{dt} e^{\mu_h t} + \mu_h N_h e^{\mu_h t} \le \beta_h e^{\mu_h t} \tag{13}$$

Integrate both sides of the inequality in equation (13) w.r.t. t, we have:

$$N_h e^{\mu_h t} \le \frac{\beta_h e^{\mu_h t}}{\mu_h} + c$$

Where c is a constant

$$\Rightarrow N_h \le \frac{\beta_h}{\mu_h} + c e^{\mu_h t} \tag{14}$$

Using the initial conditions at t=0, $N_h(t) = N_h(0)$, we have

$$N_h(0) \le \frac{\beta_h}{\mu_h} + c$$

$$\Rightarrow N_h \le \frac{\beta_h}{\mu_h} + \left(N_h(0) - \frac{\beta_h}{\mu_h} \right) e^{\mu_h t}$$

$$0 \le N_h \le \frac{\beta_h}{\mu_h} \text{ as } t \to \infty$$

Thus, the human population is given as:

$$N_h \le \frac{\beta_h}{\mu_h} \tag{15}$$

Hence;

$$\pi_h = \left\{ (S_h, E_h, I_h, R_{rh}, R_h) \in \mathbb{R}_+^5, S_h > 0, E_h \ge 0, I_h \ge 0, R_{rh} \ge 0, R_h \ge 0, N_h \le \frac{\beta_h}{\mu_h} \right\} \tag{16}$$

Similarly, the feasible solutions of the vector (mosquito) population is given as

$$\pi_v = (S_v, E_v, I_v) \in \mathbb{R}_+^3; S_v > 0; E_v \ge 0; I_v \ge 0; N_v \le \frac{\beta_v}{\mu_v} \tag{17}$$

$$\pi = (S_h, E_h, I_h, R_{rh}, R_h, S_v, E_v, I_v) \in \mathbb{R}_+^8, (S_h, S_v) > 0, (E_h, I_h, R_{rh}, R_h, E_v, I_v) \ge 0, \left(N_h \le \frac{\beta_h}{\mu_h}, N_v \le \frac{\beta_v}{\mu_v} \right) \tag{18}$$

Disease-free equilibrium point

Malaria free equilibrium occurs when there is no malaria in the human population and plasmodium parasite is not present in the mosquito population. The equilibrium point is obtained by setting the system of equation (3) to zero. Since there is absence of malaria in the population, i.e. $E_h = 0$, $I_h = 0$, $R_{rh} = 0$, $E_v = 0$, $I_v = 0$, $R_h = 0$ (since there will be no malaria to recover from), then the system of equation (1) becomes;

$$\frac{dS_h}{dt} = \beta_h + \sigma R_h - (\mu_h + \lambda) S_h - b S_h I_v = 0 \tag{19}$$

$$\frac{dS_v}{dt} = \beta_v + \gamma S_v I_v - (\mu_v + \rho + \phi + \wedge) S_v = 0 \tag{20}$$

Substitute $I_v = 0$, $R_h = 0$ in equation (19), we have;

$$\beta_h - (\mu_h + \lambda) S_h = 0$$

$$\beta_h = (\mu_h + \lambda) S_h$$
$$S_h = \frac{\beta_h}{(\mu_h + \lambda)} \quad (21)$$

Also, substitute $I_h = 0$ into equation (20);

$$\beta_v - (\mu_v + \rho + \phi + \wedge) S_v = 0$$
$$\beta_v = (\mu_v + \rho + \phi + \wedge) S_v$$
$$S_v = \frac{\beta_v}{(\mu_v + \rho + \phi + \wedge)} \quad (22)$$

Similarly, when $E_h = I_h = R_{rh} = 0$, $E_v = I_v = 0$, $R_h = 0$, then the remaining equations becomes;

$$E_h^0 = 0, I_h^0 = 0, R_{rh}^0 = 0, E_v^0 = 0, I_v^0 = 0, R_h^0 = 0$$

Therefore, the disease-free equilibrium point of the model is given as:

$$E^0 = \left(S_h^0, E_h^0, I_h^0, R_{rh}^0, R_h^0, S_v^0, E_v^0, I_v^0\right)$$
$$= \left(\frac{\beta_h}{\mu_h}, 0, 0, 0, 0, \frac{\beta_v}{(\mu_v + \rho + \phi + \wedge)}, 0, 0\right) \quad (23)$$

Thus, E^0 shows the state in which there is absence of malaria.

Stability of disease-free equilibrium

Disease-free equilibrium of the system of equation (3) is asymptotically stable if $R_0<1$. And the stability of E^0 can be tested using the eigenvalues of the Jacobian matrix obtained at disease-free equilibrium;

$$J(E^0) = \begin{bmatrix} -(\mu_h + \lambda) & 0 & 0 & 0 & \sigma & 0 & 0 & -\frac{b\beta_h}{(\mu_h + \lambda)} \\ 0 & -(\mu_h + \psi) & 0 & 0 & 0 & 0 & 0 & \frac{b\beta_h}{(\mu_h + \lambda)} \\ 0 & \psi & -b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & -k & 0 & 0 & 0 & 0 \\ \lambda & 0 & \alpha & \omega & -(\mu_v + \sigma) & 0 & 0 & 0 \\ 0 & 0 & -\frac{\gamma\beta_v}{(\mu_v + \rho + \phi + \wedge)} & 0 & 0 & p & 0 & 0 \\ 0 & 0 & \frac{\gamma\beta_v}{(\mu_v + \rho + \phi + \wedge)} & 0 & 0 & 0 & -(\mu_v + \tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau & -\mu_v \end{bmatrix} \quad (24)$$

where;

$b = (\mu_h + \alpha + \theta + + \delta)$, $k = (\mu_h + \omega + \delta)$, $p = -(\mu_v + \rho + \phi + \Lambda)$

Endemic Equilibrium Point is a state at which malaria cannot be totally eliminated (i.e. malaria still persists) in the population.

The Endemic Equilibrium point is derived by equating the right-hand side of equations (3) to zero and solving simultaneously

$$\beta_h + \sigma R_h - (\mu_h + \lambda) S_s - b S_h I_v = 0$$
$$b S_h I_v - (\mu_h + \psi) E_h = 0$$
$$\psi E_h - (\mu_h + \alpha + \theta + \delta) I_h = 0$$
$$\theta I_h - (\mu_h + \omega + \delta) R_{rh} = 0 \quad (25)$$
$$\omega R_{rh} + \alpha I_h - (\sigma + \mu_h) R_h + \lambda S_n = 0$$
$$\beta_v - \gamma S_v I_h - (\mu_v + \rho + \phi + \wedge) S_v = 0$$
$$\gamma S_v I_h - (\mu_v + \tau) E_v = 0$$
$$\tau E_v - \mu_v I_v = 0$$

Solving the above equations simultaneously to give the following new equations

$$S_h^* = \frac{\beta_h + \sigma R_h^*}{(b I_v^* + \mu_h + \lambda)}$$
$$E_h^* = \frac{b S_h^* I_v^*}{(\mu_h + \psi)}$$

$$
\begin{aligned}
I_h^* &= \frac{\psi E_h^*}{(\mu_h+\alpha+\theta+\delta)} \\
R_{rh}^* &= \frac{\theta I_h^*}{(\mu_h+\omega+\delta)} \\
R_h^* &= \frac{\omega R_{rh}^* + \alpha I_h^* + \lambda S_h^*}{(\sigma+\mu_h)} \\
S_v^* &= \frac{\beta_v}{(\gamma I_h^* + (\mu_v+\rho+\phi+\wedge))} \\
E_v^* &= \frac{\gamma S_v^* I_h^*}{(\mu_v+\tau)} \\
I_v^* &= \frac{\tau E_v^*}{\mu_v}
\end{aligned}
\tag{26}
$$

Basic reproduction number

The Basic reproduction number is obtained by using a next-generation matrix as follows

From the above system, F_i and V_i are defined as

F_i is the rate of appearance of new infections in compartment *i*.

V_i is the transfer of individuals into compartment *i*.

For the computation of *F*,

$$
F_i = \begin{bmatrix} bS_h I_v \\ 0 \\ 0 \\ \gamma S_v I_h \\ 0 \end{bmatrix}
\tag{27}
$$

$$
F = \begin{bmatrix} 0 & 0 & 0 & 0 & bS_h \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma S_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\tag{28}
$$

For the computation of *V*,

$$
V_i = \begin{bmatrix} -(\mu_h+\psi+\wedge)E_h \\ \psi E_h - (\alpha+\theta+\mu_h+\delta)I_h \\ \theta I_h - (\mu_h+\delta+\pi)I_{rh} \\ -(\mu_v+\rho+\phi+\tau)E_v \\ \tau E_v - (\mu_v+\rho+\phi)I_v \end{bmatrix}
\tag{29}
$$

$$
V = \begin{bmatrix} -(\mu_h+\psi) & 0 & 0 & 0 & 0 \\ \psi & -(\alpha+\theta+\mu_h+\delta) & 0 & 0 & 0 \\ 0 & \theta & -(\mu_h+\delta+\pi) & 0 & 0 \\ 0 & 0 & 0 & -(\mu_v+\tau) & 0 \\ 0 & 0 & 0 & \tau & -\mu_v \end{bmatrix}
\tag{30}
$$

Then,

$$
V^{-1} = \begin{bmatrix} \frac{1}{-(\mu_h+\psi)} & 0 & 0 & 0 & 0 \\ -\frac{\psi}{(\mu_h+\psi)(\alpha+\theta+\mu_h+\delta)} & \frac{1}{-(\alpha+\theta+\mu_h+\delta)} & 0 & 0 & 0 \\ \frac{\psi\theta}{-(\mu_h+\psi)(\alpha+\theta+\mu_h+\delta)(\mu_h+\delta+\pi)} & -\frac{\theta}{(\alpha+\theta+\mu_h+\delta)(\mu_h+\delta+\pi)} & \frac{1}{-(\mu_h+\delta+\pi)} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{-(\mu_v+\tau)} & 0 \\ 0 & 0 & 0 & \tau & 1 \end{bmatrix}
\tag{31}
$$

The next generation matrix FV^{-1} is

$$FV^{-1}=\begin{bmatrix}0&0&0&0&bS_h\\0&0&0&0&0\\0&0&0&0&0\\0&\gamma S_v&0&0&0\\0&0&0&0&0\end{bmatrix}*\begin{bmatrix}\frac{1}{-(\mu_h+\psi)}&0&0&0&0\\-\frac{\psi}{(\mu_h+\psi)(\alpha+\theta+\mu_h+\delta)}&\frac{1}{-(\alpha+\theta+\mu_h+\delta)}&0&0&0\\\frac{\psi\theta}{-(\mu_h+\psi)(\alpha+\theta+\mu_h+\delta)(\mu_h+\delta+\pi)}&-\frac{\theta}{(\alpha+\theta+\mu_h+\delta)(\mu_h+\delta+\pi)}&\frac{1}{-(\mu_h+\delta+\pi)}&0&0\\0&0&0&\frac{1}{-(\mu_v+\tau)}&0\\0&0&0&-\frac{\tau}{\mu_v(\mu_v+\tau)}&\frac{1}{-\mu_v}\end{bmatrix} \quad (32)$$

$$\begin{bmatrix}0&0&0&-\frac{\tau bS_h}{(\mu_v+\tau)\mu_v}&-\frac{bS_h}{\mu_v}\\0&0&0&0&0\\0&0&0&0&0\\-\frac{\gamma\psi S_v}{(\mu_h+\psi)(\alpha+\theta+\mu_h+\delta)}&-\frac{1}{(\alpha+\theta+\mu_h+\delta)}&0&0&0\\0&0&0&0&0\end{bmatrix}$$

From the above, the eigenvalues are given as:

$$\lambda_1=\lambda_2=\lambda_3=0,$$

$$\lambda_4=-\sqrt{\frac{\tau b\beta_h\gamma\psi\beta_v}{(\mu_h+\lambda)(\mu_h+\psi)(\mu_h+\alpha+\theta+\delta)(\mu_v)(\mu_v+\tau)(\mu_v+\rho+\phi+\wedge)}}$$

$$\lambda_5=\sqrt{\frac{\tau b\beta_h\gamma\psi\beta_v}{(\mu_h+\lambda)(\mu_h+\psi)(\mu_h+\alpha+\theta+\delta)(\mu_v)(\mu_v+\tau)(\mu_v+\rho+\phi+\wedge)}}$$

It can be seen that the dominant eigenvalue is

$$\lambda_5=\sqrt{\frac{\tau b\beta_h\gamma\psi\beta_v}{(\mu_h+\lambda)(\mu_h+\psi)(\mu_h+\alpha+\theta+\delta)(\mu_v)(\mu_v+\tau)(\mu_v+\rho+\phi+\wedge)}}$$

The Reproductive ratio R_0 is given as;

$$\sqrt{\frac{\tau bS_h}{(\mu_v)(\mu_v+\tau)}}\sqrt{\frac{\gamma\psi S_v}{(\mu_h+\psi)(\mu_h+\alpha+\theta+\delta)}}$$

$$R_0=\sqrt{\frac{\tau b\beta_h\gamma\psi\beta_v}{(\mu_h+\lambda)(\mu_h+\psi)(\mu_h+\alpha+\theta+\delta)(\mu_v)(\mu_v+\tau)(\mu_v+\rho+\phi+\wedge)}}$$

Basic reproduction number for the combination of interventions for the proposed model

From Table 2, the interventions combination is only effective when R_0<1

Numerical simulation results

Numerical result: The simulations were carried out using Mathematica's ParametricNDSolve function and the parameters were obtained from literature as shown in Table 3, 4.

Parameter values

Figure 2 shows the effects of using the interventions and the effects of not using the interventions in reducing malaria transmission in humans. It also shows that the usage of the interventions helps to reduce the number of both susceptible humans and infected humans.

Figure 3 shows the effects of using the interventions and the effects of not using the interventions in reducing the breeding of vectors (mosquitoes) in the population. It also shows that the usage of the interventions helps to reduce the number of susceptible vectors.

Table 2: Reproduction numbers with interventions combination.

Interventions	Reproduction Number
Malaria vaccines and anti-malaria drugs	0.6048
Malaria vaccines and LLINS	2.4996
Malaria vaccines and IRS	2.8435
Malaria vaccines and Environmental sanitation	3.8039
Anti-malaria drugs, malaria vaccines and LLINS	0.1823
Anti-malaria drugs, malaria vaccines and Environmental sanitation	0.277
Anti-malaria drugs, malaria vaccines and IRS	0.2074
Malaria vaccines, IRS and Environmental sanitation	2.3686
Malaria vaccines, LLINS and Environmental sanitation	2.1586
Anti-malaria drugs, Malaria vaccines, LLINS and IRS	0.1406
Environmental sanitation, IRS, LLINS and Malaria vaccines	0.1758
Anti-malaria drugs, Malaria vaccines, IRS and Environmental sanitation	0.1728
Anti-malaria drugs, Environmental sanitation, Malaria vaccines and LLINS	0.1575
Malaria vaccines, Anti-malaria drugs, IRS, LLINS and Environmental sanitation	0.1282

The interventions combination is only effective when $R_0<1$

Table 3: Parameter values of the proposed model obtained from literature.

Parameters	Values	References
β_h	1.2	Adesoye et al. [11]
b	0.0025	Kipkirui et al. [12]
γ	0.415	Kipkirui et al. [12]
ψ	0.058 per day	Kuddus et al. [13]
α	0.4	Resmawan [14]
σ	0.00014	Resmawan [14]
δ	0.0004 per day	Kipkirui et al. [12]
μ_h	0.00004 per day	Kipkirui et al. [12]
β_v	6.000	Kipkirui et al. [12]
μ_v	0.04 per day	Kipkirui et al. [12]
θ	0.0017	Titus et al. [15]
ω	0.5	Resmawan [14]
λ	0.5	Resmawan [14]
τ	0.0833	Kipkirui et al. [12]
$\rho, \emptyset, \Lambda$	0.3, 0.4, 0.15	Kipkirui et al. [12], Dereje et al. [16], Dereje et al. [16]

Table 4: Changes in the parameter values to show the effectiveness of the interventions on the basic reproduction number.

Parameter Values	Reproduction Number
λ=0.5, α=0.4, Λ=0.15, ρ=0.3, ∅=0.4	0.1282
λ=0.6, α=0.5, Λ=0.25, ρ=0.4, ∅=0.5	0.0906
λ=0.7, α=0.6, Λ=0.35, ρ=0.5, ∅=0.6	0.0684
λ=0.8, α=0.7, Λ=0.45, ρ=0.6, ∅=0.7	0.0541
λ=0.9, α=0.8, Λ=0.55, ρ=0.7, ∅=0.8	0.0441

Conclusion

This study proposed a modified SEIRR-SEI mathematical model to study the transmission and breeding of mosquitoes for the elimination of malaria in Nigeria.

The disease model is mathematically based on solutions' positivity, boundedness, equilibrium points, and local stability justifications.

The numerical simulation results of the analysis showed that the use of Long-Lasting Insecticides-treated bed Nets (LLINS), Indoor Residual Spraying (IRS), Artemisinin-based Combination Therapies (ACTs), Malaria Vaccines (MV), and Environmental Sanitation (ES) practises as interventions are effective in reducing the breeding of mosquitoes for malaria elimination in the population.

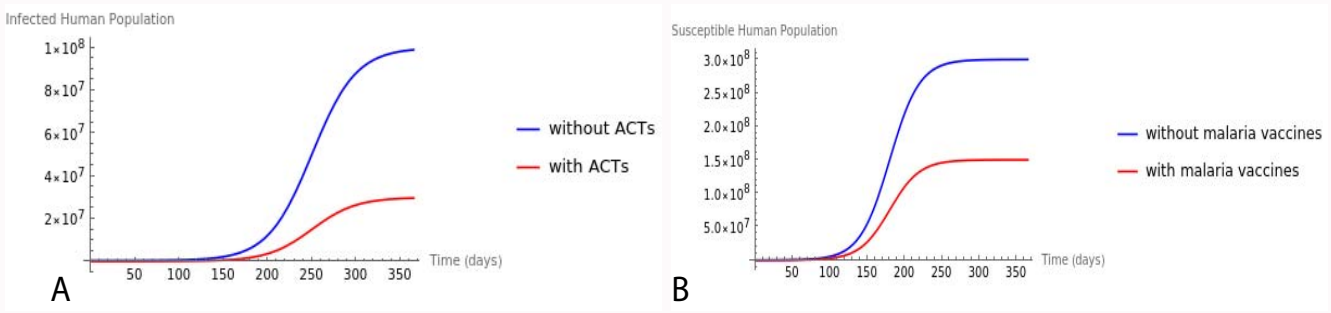

Figure 2: (A, B) Simulating the impact of Artemisinin-based Combination Therapies (ACTs), Malaria Vaccine (MV) at the rate of 0.4 and 0.5 respectively.

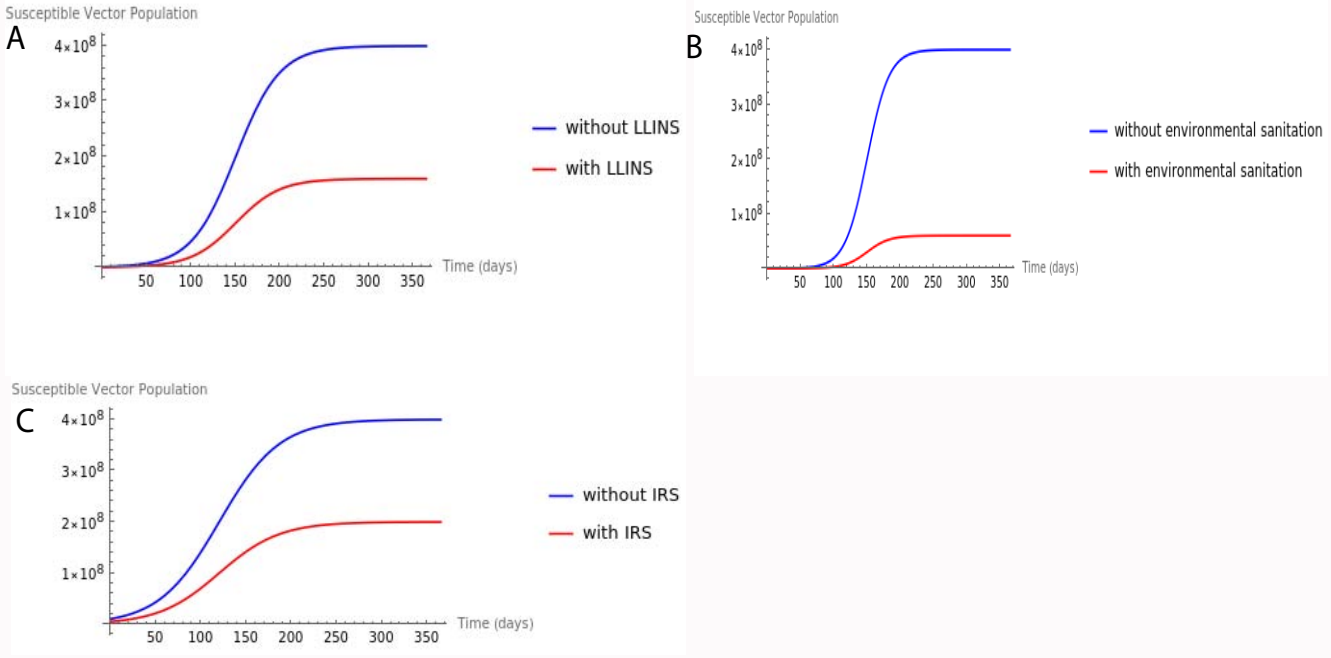

Figure 3: (A-C) Simulating the Impact of Long-Lasting Insecticide Treated Nets (LLINS), Environmental Sanitation (ES), Indoor Residual Spraying (IRS) at the rate of 0.4, 0.15, 0.3 respectively.

The basic reproduction number of the model SEIRR-SEI estimated using the next-generation matrix showed that malaria infection will eventually be reduced in the population. Also, with proper and efficient combination of all the five interventions considered in the model, the population of malaria will reduce in the population and malaria infection will eventually be eliminated in Nigeria.

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